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Critical conditions for shear localization in thermoviscoplastic materials are obtained in closed form for idealized models of simple shearing deformations. The idealizations, which include the neglect of heat conduction, inertia, and elasticity, are viewed as quite acceptable for many applications in which shear bands occur. Explicit results obtained for the idealized, but fully nonlinear problem show the roles of strain rate sensitivity, strain hardening, and initial imperfection on the localization behavior. Numerical solutions for two steels

are shown to exhibit the principal features reported for torsional Kolsky bar experiments on these steels. Mathematically exact critical conditions obtained for the fully nonlinear problem are compared with critical conditions obtained by means of linear perturbation analysis. Use of relative changes instead of absolute changes in the linear perturbation analysis gives better agreement with the predictions of the fully nonlinear analysis.

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ANALYTICAL CHARACTERIZATION OF SHEAR LOCALIZATION IN THERMOVISCOPLASTIC MATERIALS

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Abstract

Critical conditions for shear localization in thermoviscoplastic materials are obtained in closed form for idealized models of simple shearing deformations. The idealizations, which include the neglect of heat conduction, inertia, and elasticity, are viewed as quite acceptable for many applications in which shear bands occur. Explicit results obtained for the idealized, but fully nonlinear problem show the roles of strain rate sensitivity, strain hardening, and initial imperfection on the localization behavior. Numerical solutions for two steels are shown to exhibit the principal features reported for torsional Kolsky bar experiments on these steels. Mathematically exact critical conditions obtained for the fully nonlinear problem are compared with critical conditions obtained by means of linear perturbation analysis. Use of relative changes instead of absolute changes in the linear perturbation analysis gives better agreement with the predictions of the fully nonlinear analysis.

1. Introduction

Shear instabilities in the form of shear bands are commonly observed in metals and polymers subjected to large deformations. The formation of a shear band is often an immediate precursor to rupture of the material. Even when rupture does not occur, the development of shear bands generally reduces the performance of the material. Thus, improved understanding of shear band formation is critical to the development of improved materials and components made from these materials.

Shear bands can be divided into two types: those in which thermal softening plays a negligible role in their formation and those in which thermal softening plays a primary role. In the former case the shear bands, sometimes called isothermal shear bands, form as a result of strain softening due, for example, to material damage, to the development of soft textures, or to phase transformations. In the latter case the shear bands, often called adiabatic shear bands, form as the result of an autocatalytic process: an increase in strain rate in a weaker zone causes a local increase in temperature which in turn, for a thermal softening material, causes a further increase in strain rate.

In this paper we consider both types of shear bands. We limit our attention to simple shearing deformations. Two fundamental questions regarding the critical conditions for shear band formation are addressed.

1. For a given constitutive law, will shear localization occur for a sufficiently large shear?
2. If so, what is the critical shear γ^c, outside of the shear band, for which the catastrophic process occurs?

As background for this study we note that an analysis of the stability of homogeneous simple shearing deformations has been presented by CLIFTON (1978)

for quasi-static deformations and BAI (1982) for dynamic deformations. They used a classical, linear perturbation analysis in which the coefficients in the linear differential equations for the perturbations were assumed to vary sufficiently slowly that these variations could be neglected in estimating the rate of growth or decay of fluctuations from the homogeneous solution. This procedure determines a critical strain at which fluctuations begin to grow; however, this initial growth may or may not lead to instability depending on the neglected effects of the time dependence of the coefficients and the nonlinearity of the complete system of equations. MOLINARI and CLIFTON (1983) and MOLINARI (1984, 1985) have presented some analytical solutions of the fully nonlinear problem under quasi-static and adiabatic (no heat conduction) conditions. With these solutions available for measuring the reliability of more simple approaches for determining the onset of instability, MOLINARI (1985), and FRESSENGEAS and MOLINARI (1987) developed a so-called relative linear perturbation analysis that accounts, in part, for the non-steadiness of the homogeneous solution by linearizing in the relative perturbation defined as the perturbation divided by the corresponding unperturbed quantity. This approach has been shown to give predictions, as to whether or not shear bands will form, that are more in agreement with the fully nonlinear theory than are predictions based on classical linear perturbation analysis. DAFERMOS and HSIAO (1983) obtained a priori estimates of the asymptotic behavior of the solution of the nonlinear problem (including inertia, but not heat conduction) for the case of a Newtonian fluid with temperature-dependent viscosity. TZAVARAS (1984) extended these results to the case of non-Newtonian fluids with temperature-dependent viscosities.

Numerical solutions of the fully nonlinear system of equations have been presented by several authors: SHAWKI, CLIFTON and MAJDA (1983), SHAWKI

(1986), WRIGHT and BATRA (1985), MOLINARI (1985). From these solutions one can conclude that dynamical effects and heat conduction are relatively unimportant for steel specimens, with lengths of 5-10 mm, subjected to shearing rates of $10^3 s^{-1}$ as in the torsional Kolsky bar experiments of COSTIN, CRISMAN, HAWLEY and DUFFY (1979), and HARTLEY (1986). Thus, in this paper we neglect dynamical effects and heat conduction in order to present an analytical approach to the fully nonlinear problem of thermoviscoplastic localization in simple shear. Our aim is to obtain simple analytical formulae for determining whether or not a shear strain localization instability will occur and, if so, the critical strain γ^c at which the localization becomes catastrophic. The boundary conditions will, in some cases, be general whereas in others they will be restricted to a constant imposed shear stress or a constant imposed velocity. Isothermal shear bands are considered in Section 2 and adiabatic shear bands are considered in Sections 3 and 4.

2. Isothermal problem

We consider a simple shearing deformation of strain hardening material with strain-rate sensitivity. For illustration, we consider the following constitutive law:

$$\tau = f(\gamma)\, \dot{\gamma}^m \qquad (m > o) \qquad (1)$$

where τ is the shear stress, γ is the shear strain and $\dot{\gamma}$ is the shear rate. The function $f(\gamma)$ takes account of the strain hardening. This function is not necessarily monotonically increasing in order to account for possible strain softening.

Suppose that, for a constant applied strain rate $\dot{\gamma}$, the shear stress τ passes through a maximum. Will strain localization occur? By localization we mean that in some narrow region, the strain becomes much larger than elsewhere. More precisely, we can define two types of localization.

L_P localization

If the heterogeneity of the solution is growing so that at time t, a region R (necessarily narrow) exists where $\gamma/\bar{\gamma} > P$ with $\bar{\gamma}$ representing the average deformation and P being a large number, then L_P localization of the deformation is said to occur.

L_∞ localization

If for every point A different from B, the ratio $\dot{\gamma}_B/\dot{\gamma}_A$ tends to infinity with increasing time, then L_∞ localization of the deformation at the point B is said to occur.

The analysis of localization in this section is performed in two different ways. First we derive an analytical solution of the fully nonlinear problem. Then an absolute and a relative linear perturbation analysis

are performed and the corresponding predictions are compared with the exact solution.

2.1 The nonlinear theory

We consider a slab with a geometrical defect. The width $\ell(y)$ is nonuniform as shown in Figure 1. Using the same approach as HUTCHINSON and NEALE (1977) for the uniaxial tension of a bar, we get from the equilibrium equation written at two different points A and B:

$$\ell_A \tau_A = \ell_A\, f(\gamma_A)(\dot{\gamma}_A)^m = \ell_B \tau_B = \ell_B\, f(\gamma_B)(\dot{\gamma}_B)^m \, . \tag{2}$$

Taking the power 1/m of each term, we get after integration:

$$\ell_A^{1/m} \int_{\gamma_A^o}^{\gamma_A} (f(\zeta))^{1/m}\, d\zeta = \ell_B^{1/m} \int_{\gamma_B^o}^{\gamma_B} (f(\zeta))^{1/m}\, d\zeta \tag{3}$$

where γ_A^o and γ_B^o are the initial strains at points A and B. If $(f(\zeta))^{1/m}$ is integrable at infinity, then the values of the integrals are finite. While maintaining the equality (3), let γ_B and γ_A he increased until the strain becomes infinite at one of the two points, say B. Then there exists for each γ_A^o a finite strain γ_A for which Eqn. (3) is satisfied. Hence, we have L_∞ localization of strain if and only if the function $(f(\zeta))^{1/m}$ is integrable at infinity.

Let us consider two examples:

(a) <u>$f(\zeta)\to 0$ as $\zeta\to\infty$:</u>

Assume that $f(\zeta)$ has power law behavior at infinity of the form

$$f(\zeta) \sim a\zeta^{-p} \text{ as } \zeta\to\infty \tag{4}$$

where a and p are positive constants. Then, from the integrability condition (3), the deformation exhibits L_∞ localization if and only if

$$-p + m < 0 \quad . \tag{5}$$

This condition illustrates the stabilizing effect of the strain-rate sensitivity for $m > 0$. Even if the material is strain softening ($p > 0$), localization will occur only if m is sufficiently small ($m < p$) (Figure 2c).

If localization does occur, the critical localization strain can be easily calculated. Let us consider the following function h defined for each point M of the slab:

$$h(M) = \ell_M^{1/m} \int_{\gamma_M^o}^{\infty} (f(\zeta))^{1/m} \, d\zeta. \tag{6}$$

Localization of strain will occur at the point B where the function h is a minimum. The critical localization strain γ_A^c at a point A is given by the implicit equation.

$$\ell_A^{1/m} \int_{\gamma_A^o}^{\gamma_A^c} (f(\zeta))^{1/m} \, d\zeta = \ell_B^{1/m} \int_{\gamma_B^o}^{\infty} (f(\zeta))^{1/m} \, d\zeta. \tag{7}$$

In the particular case where the initial strain γ^o is uniform the localization will occur at the points B where the width ℓ_B is minimum.

For $m - p > 0$, L_∞ localization does not occur. Indeed, it is readily shown in Appendix A that

$$\lim_{\gamma_A \to \infty} (\gamma_B/\gamma_A) = (\ell_A/\ell_B)^{1/(m-p)} \tag{8}$$

This quantity tends to infinity as $m - p \to 0^+$. Then, according to our definition, L_p localization occurs for $m - p$ small enough (Figure 2b).

(b) $f(\zeta) \to L$ as $\zeta \to \infty$:

In the second example, we consider a function $f(\zeta)$ for which the limit at infinity exists

$$\lim_{\zeta \to \infty} f(\zeta) = L > 0 \; . \tag{9}$$

Then the result in appendix A combined with Equation (3) shows that:

$$\begin{aligned}(\ell_A/\ell_B)^{1/m} &= \int_{\gamma_B^o}^{\gamma_B} f(\zeta)^{1/m}\, d\zeta \Big/ \int_{\gamma_A^o}^{\gamma_A} f(\zeta)^{1/m}\, d\zeta \sim (L^{1/m}\,\gamma_B) \,/\, (L^{1/m}\,\gamma_A) \\ &= \gamma_B/\gamma_A \text{ as } \gamma_A \to \infty \; .\end{aligned} \tag{10}$$

The shape of the curve $\gamma_A \to \gamma_B$ is shown in Figure 3 for $\gamma_B \geqslant \gamma_A$. As γ_B passes through the strain at which $f(\gamma_B)$ is a maximum the ratio γ_B/γ_A increases strongly. It is then possible that γ_B/γ_A takes on large values, say $\geqslant$ P. Thus, L_P localization may occur just after $f(\gamma_B)$ passes through a maximum. As the deformation continues, the ratio γ_B/γ_A decreases and tends to $\gamma_B/\gamma_A|_\infty = (\ell_A/\ell_B)^{1/m}$. If $(\ell_A/\ell_B)^{1/m}$ is not large enough, L_P localization does not occur as $\gamma \to \infty$. Therefore, L_P localization may occur after the maximum of $f(\gamma)$ is reached, and disappear for larger deformations.

Strain rate sensitivity has a strong stabilizing effect on the asymptotic behavior of the solution. For example, consider a 1% geometrical defect:

$$\ell_B/\ell_A = 0.99$$

Values of $(\gamma_B/\gamma_A)_\infty = (\ell_A/\ell_B)^{1/m}$ are given in Table 1 for different values of m

Table 1:

Influence of Strain Rate Sensitivity on the Asymptotic Behavior of Plastic Flow (geometrical defect: $\ell_B/\ell_A = 0.99$)

m	0.2	0.1	0.01	0.006	0.001
$(\gamma_B/\gamma_A)_\infty$	1.052	1.106	2.732	7.464	2.316×10^4

A value $m = 0.01$ is sufficient to prevent pronounced localization as $\gamma \to \infty$. For small values of m (say $m < 0.005$) L_P localization occurs as $\gamma \to \infty$.

These examples illustrate that a maximum in the stress-strain curve does not lead necessarily to the localization of plastic flow.

2.2 Linear Perturbation Analysis

It is interesting to compare the results of the fully nonlinear theory to the predictions of a linear stability analysis. For this comparison consider a block of uniform thickness $\ell(y) = \ell_o$ undergoing homogeneous simple shearing deformation $\gamma_o(t)$. Let $\delta\gamma = \gamma(y,t) - \gamma_o(t)$ be the difference between the shear strain $\gamma(y,t)$ for the same block subjected to the same boundary conditions, but having a fluctuation in strain and strain rate beginning at some time t_o. Using the constitutive law (1) and considering the problem as quasistatic ($\delta \dot{\tau} = o$) we obtain:

$$\frac{\delta\dot{\gamma}}{\delta\gamma} = - \frac{f'(\gamma_o)}{f\,(\gamma_o)} \frac{\dot{\gamma}_o}{m} \tag{9}$$

when $\delta\gamma$ is sufficiently small. Equation (9) shows that, at least initially, the strain difference $\delta\gamma$ grows when $f'(\gamma) < 0$ i.e. when strain softening occurs. If strain hardening occurs, i.e $f'(\gamma) > 0$, then the strain difference

$\delta\gamma$ decays initially. Because the right side of Equation (9) is independent of the coordinate y, the growth or decay of the strain difference is such that the ratio of the strains $\delta\gamma_A$ and $\delta\gamma_B$ at points A and B remains constant. Thus, Equation (9) provides no information on strain localization.

From this linear analysis we see that whether small perturbations of a homogeneous shearing deformation are expected to grow or decay initially depends only on the sign of $f'(\gamma_o)$, the slope of the stress-strain curve at a constant strain γ_o. However, from the nonlinear analysis, we know that whether or not localization will occur is not governed by the value of $f'(\gamma_o)$, but by the strain rate sensitivity parameter m and the behavior of $f(\gamma)$ as $\gamma\to\infty$. The condition $f'(\gamma) < 0$ is a necessary and sufficient condition for the initial growth of perturbations of a homogeneous deformation whereas $f'(\gamma) < 0$ for γ greater than some critical value γ_c is only a necessary condition for localization to occur. This tendency for the linear perturbation analysis based on (9) to predict the growth of small perturbations under relatively weak restrictions on the constitutive equations can be partially offset by considering the relative perturbation

$$\Delta\gamma = \frac{\delta\gamma}{\gamma_o} \quad . \tag{10}$$

The relative perturbation $\Delta\gamma$ tends to grow more slowly than the absolute perturbation $\delta\gamma$ and may even decay as the perturbation grows.

From (9) and (10) we obtain after logarithmic differentiation

$$\frac{\dot{\Delta\gamma}}{\Delta\gamma} = \frac{\dot{\delta\gamma}}{\delta\gamma} - \frac{\dot{\gamma}_o}{\gamma_o} = - \left[\frac{\gamma\, f'\,(\gamma_o)}{m\, f\,(\gamma_o)} + 1 \right] \frac{\dot{\gamma}_o}{\gamma_o} \tag{11}$$

Integration of (11) gives

$$\Delta\gamma = K \frac{f(\gamma_0)^{-1/m}}{\gamma_0} \quad . \tag{12}$$

If $f(\gamma_0)$ has the behavior (4) for large values of γ_0, then as $\gamma_0 \to \infty$ the relative perturbation $\Delta\gamma$ becomes unbounded for $-p + m < 0$ and approaches zero for $-p + m > 0$. These conditions are, respectively, the same as the critical conditions (See example (a) of the previous section) for L_p-localization to occur or not. This parallellism between predictions of the linear relative perturbation analysis and the exact results for the nonlinear theory suggests that linear relative perturbation analysis may be more widely useful in predicting the stability of deformations than is the commonly used linear perturbation analysis represented by Equation (9). However, we emphasize that the localization analysis in the nonlinear theory and the linear relative perturbation analysis address different problems and there is no a priori reason to expect that the critical conditions for L_∞-localization are, in general, the same as the critical conditions for predicted unbounded growth of a relative perturbation.

3. Adiabatic Case

We consider next the influence of temperature on localization. As discussed in the introduction, the deformation is assumed to be adiabatic and quasistatic. We consider the constitutive equation

$$\tau = \tau\ (\gamma,\ \dot{\gamma},\ \theta), \tag{13}$$

the equation of equilibrium

$$\ell(y)\ \tau(y,t) = \ell(h)\ \tau(h,t), \tag{14}$$

the compatibility equation

$$\dot{\gamma} = \frac{\partial v}{\partial y}\ , \tag{15}$$

and the energy equation

$$\rho C\ \frac{\partial \theta}{\partial t} = \ \beta\ \tau\ \dot{\gamma}\ . \tag{16}$$

In these equations ρ is the mass density, C is the heat capacity per unit mass, θ is the absolute temperature, v is the particle velocity, and β is the Taylor-Quinney coefficient which characterizes the fraction of plastic work that is converted into heat; usually β is taken constant and equal to 0.9. Equations (13)-(16) constitute four equations in the four unknowns γ,θ,τ,v.

In the following, we present a discussion of localization for different constitutive laws and different boundary conditions. We consider the cases of constant velocity boundary conditions:

$$v(0,t) = 0$$
$$v(h,t) = v_o \tag{17}$$

or constant stress boundary conditions

$$\ell(h)\tau(h,t) = \ell(0)\tau(0,t) = \text{const.} \tag{18}$$

3.1 Materials Without Strain-Hardening

An exact solution of the fully nonlinear problem has been presented by MOLINARI and CLIFTON (1983) for the case in which the material is not strain hardening and Eqn. (13) has the form

$$\tau = \mu(\theta)\,\dot{\gamma}^m\ . \tag{19}$$

In order to obtain this exact solution we write Eqns. (14) and (16) at two different points A and B. Substitutions of (19) into (14) and use of (16) to eliminate $\dot{\gamma}_A/\dot{\gamma}_B$ gives

$$\ell_A^{(m+1)/m}\mu(\theta_A)^{1/m}\ d\theta_A = \ell_B^{(m+1)/m}\mu(\theta_B)^{1/m}\ d\theta_B \tag{20}$$

which, after integration, becomes

$$\ell_A^{(m+1)/m}\int_{\theta_A^o}^{\theta_A}\mu(\zeta)^{1/m}\,d\zeta = \ell_B^{(m+1)/m}\int_{\theta_B^o}^{\theta_B}\mu(\zeta)^{1/m}\,d\zeta \tag{21}$$

where θ_A^o and θ_B^o are the initial temperatures at points A and B.

From (21) it appears that L_∞ localization of temperature occurs at B if, and only if, $\mu(\theta)^{1/m}$ is integrable at infinity, i.e. if there exists some K > 0 for which

$$\int_K^{\infty} \mu(\zeta)^{1/m} < + \infty \; . \tag{22}$$

Localization will occur at the point B where the following function, defined for each point M of the slab,

$$M \to \ell_M{}^{(m+1)/m} \int_{\theta_M^o}^{\infty} \mu(\zeta)^{1/m} \, d\zeta \tag{23}$$

is a minimum. At localization the temperature θ_A^c at any point A is given by

$$\ell_A^{(m+1)/m} \int_{\theta_A^o}^{\theta_A^c} \mu(\zeta)^{1/m} \, d\zeta = \ell_B^{(m+1)/m} \int_{\theta_B^o}^{\infty} \mu(\zeta)^{1/m} \, d\zeta \; . \tag{24}$$

It is easy to show that L_∞ temperature localization tends to result in strain localization. Indeed, from the equilibrium condition (15) and the constitutive law (19), we have

$$\ell_A \, \mu(\theta_A) \, \dot{\gamma}_A^m = \ell_B \, \mu(\theta_B) \dot{\gamma}_B^m \; .$$

Then, assuming temperature localization, we get

$$\lim_{\theta_A \to \theta_A^c} \left[\frac{\dot{\gamma}_B}{\dot{\gamma}_A} \right] = \lim_{\theta_A \to \theta_A^c} \left[\frac{\ell_A \, \mu(\theta_A)}{\ell_B \, \mu(\theta_B)} \right]^{1/m} = + \infty$$

since $\lim_{\theta_A \to \theta_A^c} \mu(\theta_B) = 0$ from the integrability condition (22). This L_∞ localization of the strain rate essentially ensures L_∞ localization of the strain although various pathological cases must be excluded in a rigorous analysis. We henceforth consider constitutive equations and loading conditions

for which such pathological cases are excluded.

Localization results obtained from the integrability condition (22) are summarized in Table 2 for several constitutive laws.

Table 2: Localization results for visco-plastic, thermal softening materials without strain hardening

($m > 0$, $\mu_0 > 0$, $a > 0$)

	Constitutive law	L_∞ localization
L_1	$\tau = \mu_0\, \theta^{\nu}\, \dot{\gamma}^m$	$\nu + m < 0$
L_2	$\tau = \mu_0 \exp(-\alpha\theta)\, \dot{\gamma}^m$	$\alpha > 0$
L_3	$\tau = \sup(a + b\theta, 0)\, \dot{\gamma}^m$	$b < 0$
L_4	$\tau = \mu_0 \exp(-\alpha/\theta)\, \dot{\gamma}^m$	never exhibits L_∞ localization

3.1.1 Calculation of the critical strain

In this section we obtain explicit results for the critical strain at localization. We illustrate the approach by considering the constitutive law L_2. Substituting this law in Eqn. (21), we obtain the following expression for the temperature at a point B as a function of the temperature at a point A

$$\theta_B = -\frac{m}{\alpha}\log\left[\left(\frac{\ell_A}{\ell_B}\right)^{(1+m)/m}\exp(-\alpha\theta_A/m) + C_1\right] \tag{25}$$

where

$$C_1 = \exp(-\alpha\theta_B^o/m) - \left(\frac{\ell_A}{\ell_B}\right)^{(1+m)/m}\exp(-\alpha\theta_A^o/m). \tag{26}$$

From (25), a necessary condition for localization to occur is $C_1 < 0$ since for $C_1 \geq 0$ the logarithm cannot tend to infinity. Let us identify the point B as the point where the quantity $\ell_B^{(m+1)/m}\exp(-\alpha\theta_B^o/m)$ is a minimum. Then, if the initial temperature (or the width ℓ) is non uniform, C_1 is strictly negative.

For a thermal softening material, i.e. $\alpha > 0$, the quantity $\exp(-\alpha\theta_A/m)$ decreases to zero as $\theta_A \to \infty$. For some critical temperature θ_A^c, the temperature θ_B will become infinite. From (26), this value is

$$\theta_A^c = \theta_A^o - \frac{m}{\alpha}\log\left[1 - \left(\frac{\ell_B}{\ell_A}\right)^{\frac{1+m}{m}}\exp(-\alpha(\theta_B^o - \theta_A^o)/m)\right]. \tag{27}$$

If the material is thermal hardening, i.e. $\alpha < 0$, then the term $\exp(-\alpha\theta_A/m)$ grows and, from (25), it is obvious that L_∞ localization is impossible.

Indeed, for sufficiently large θ_A, the difference

$$\theta_B - \theta_A \simeq \log \left[\left(\frac{\ell_A}{\ell_B} \right)^{(-1-m)/\alpha} \right]$$

becomes small compared to the absolute temperature, say θ_A.

To calculate the critical strain from the critical temperature we consider first the case in which a constant stress τ is applied at the boundary. Then, from the energy equation (16), θ can be calculated as a function of the strain γ

$$\theta = \frac{\beta\tau}{\rho C}\gamma + \theta^o. \tag{28}$$

The critical strain, γ_A^c, at A is obtained by substituting the critical temperature θ_A^c given by (27) into (28) to obtain

$$\gamma_A^c = -\frac{\rho C m}{\alpha\beta\tau} \log \left[1 - \left(\frac{\ell_B}{\ell_A} \right)^{\frac{1+m}{m}} \exp(-\alpha(\theta_B^o - \theta_A^o)/m) \right]. \tag{29}$$

The stabilizing effects of increased strain rate sensitivity (i.e. larger m) and decreased thermal softening (i.e. smaller α) are evident in this expression. The relative importance of geometrical and temperature defects is evident from (29) which shows that the critical strain γ_A^c is the same when (ℓ_B/ℓ_A) and $(\theta_B^o - \theta_A^o)$ belong to the locus of values for which the argument of the logarithm in (29) is a constant. For small values of

$$\Delta\ell \equiv \frac{\ell_A - \ell_B}{\ell_A} \tag{30a}$$

and

$$\Delta\theta \equiv \frac{\theta_B^o - \theta_A^o}{\theta_B^o} \tag{30b}$$

the critical shear strain γ_A^c is the same for defect amplitudes $\Delta\theta$ and $\Delta\ell$ that satisfy

$$\Delta\theta = \frac{(1 + m)}{\alpha\ \theta_o} \left[\frac{\Delta\ell}{\ell}\right] . \tag{31}$$

The critical, nominal strain at which the temperature becomes infinite at B is obtained from the integration of the critical strain $\gamma^c(y_A) \equiv \gamma_A^c$ over the height of the block. Thus, the critical, nominal strain is

$$\gamma_c = \frac{1}{h} \int_o^h \gamma^c(y_A)\ dy_A . \tag{32}$$

Numerical integration of (32) is straightforward as long as the thickness $\ell(y_A)$ varies sufficiently slowly near the point(s) B at which the strain becomes infinite.

We consider next the calculation of the critical strain for the case of the velocity boundary conditions (17). An exact solution does not appear to be possible in this case. However, a good approximate solution can be obtained for the case of weak strain-rate sensitivity (i.e. $m \ll 1$). Such weak strain rate sensitivity is commonly observed in metals at room temperature for strain rates up to 10^3 sec^{-1}. Typical values of m are of the order of $m = 0.01$. In order to obtain an approximate solution for small m we introduce the mean constant strain rate

$$\dot{\gamma}_o = V/h \tag{33}$$

For small values of m we can approximate the stress τ by

$$\tau \simeq \mu_o \exp(-\alpha\theta)\ \dot{\gamma}_o^m . \tag{34}$$

This approximation is discussed in appendix B and a numerical evaluation will be presented later. Substitution of the approximate stress (34) into the energy equation (16) leads to

$$\rho C \frac{\partial \theta}{\partial t} = \beta \; \mu_o \exp(-\alpha\theta) \; \dot{\gamma}_o^m \; \dot{\gamma} \; . \tag{35}$$

This equation can be integrated by separation of the variables θ and γ to give

$$\theta(\gamma) = \theta^o + \frac{1}{\alpha} \log \left[1 + \frac{\alpha \; \beta \; \mu_o \; \dot{\gamma}_o^m}{\rho C} \; \exp(-\alpha\theta^o) \; \gamma \right] . \tag{36}$$

where θ^o is the initial temperature.

With the relationship between θ and γ given by (36), the critical strain, γ_A^c, at A can be obtained by integration of the equilibrium equation (14). Such integration gives

$$\ell_A^{1/m} \int_o^{\gamma_A} \exp(-\alpha\theta_A(\gamma)/m) \; d\gamma = \ell_B^{1/m} \int_o^{\gamma_B} \exp(-\alpha\theta_B(\gamma)/m) \; d\gamma \tag{37}$$

with $\theta_A(\gamma)$ and $\theta_B(\gamma)$ given by (36). At localization, γ_B becomes infinite and the critical strain γ_A^c at point A becomes, for $m << 1$,

$$\gamma_A^c = \frac{\rho C}{\alpha\beta\tau_A^o} \left\{ \left[1 - \left(\frac{\ell_B}{\ell_A} \right)^{1/m} \cdot \exp(-\alpha(1-m)(\theta_B^o - \theta_A^o)/m) \right]^{-m/(1-m)} - 1 \right\} \tag{38}$$

where

$$\tau_A^o = \mu_o \; \dot{\gamma}_o^m \; \exp(-\alpha\theta_A^o)$$

is the shear stress at A in an isothermal deformation at the same strain rate. If ℓ and θ^o are both uniform, then Eqn. (38) implies that γ_A^c is infinite and localization does not occur. The critical strain decreases as (ℓ_B/ℓ_A)

decreases and $\theta_B^o - \theta_A^o$ increases. The energy measure, $\tau_A^o \gamma_A^c$, of the critical strain increases with increasing strain rate sensitivity (i.e. increasing m) and decreasing thermal softening (i.e. decreasing α). Again, the nominal critical strain is obtained by the substitution of (38) into (32). Comparison of (38) and (29) indicates that, for m << 1, the relationship (31) between equivalent temperature defects $\Delta\theta$ and geometric defects $\Delta\ell$ holds for velocity boundary conditions as well as for constant stress boundary conditions.

Further understanding of the dependence of the critical strain γ_A^c on the defect can be obtained by introducing the defect parameter

$$\chi = 1 - \left[\frac{\ell_B}{\ell_A}\right] \exp\left(-\alpha(\theta_B^o - \theta_A^o)\right). \tag{39}$$

In terms of χ the critical strain, for m << 1, is given by

$$\gamma_A^c = \frac{\rho C}{\alpha\beta\tau_A^o}\left\{[1 - (1 - \chi)^{1/m}]^{-m} - 1\right\}. \qquad (38)'$$

For sufficiently small defects (i.e. $0 < \chi << m << 1$) this expression can be approximated by

$$\gamma_A^c \approx \frac{\rho C}{\alpha\beta\tau_A^o}\left[\left[\frac{\chi}{m}\right]^{-m} - 1\right] \tag{40a}$$

or, alternatively,

$$\gamma_A^c \approx \frac{\rho C}{\alpha\beta\tau_A^o}\left[-m \log \chi + \text{const.}\right] \tag{40b}$$

where the constant is chosen such that (38) and (40b) give the same value for

γ_A^c for one value of χ in the interval of interest. From (40b) it is evident that the critical strain depends weakly on the defect parameter for sufficiently small defects. For values of m that are characteristic of steels (e.g. $0.01 \leq m \leq 0.02$) the approximate relation (40b) provides a satisfactory representation of the dependence of γ_A^c on χ over three decades of variation of χ for $\chi < 0.2$ m.

Identical calculations can be performed for a power law dependence of the flow stress on the temperature. Analogous results for the constitutive law L_1 of Table 2 are

$$\gamma_A^c = \frac{\rho c \theta_A^o}{\beta \tau_A} \left\{ \left[1 - \left(\frac{\ell_B}{\ell_A} \right)^{\frac{1+m}{m}} \left(\frac{\theta_B^o}{\theta_A^o} \right)^{\frac{\nu+m}{m}} \right]^{\frac{m}{\nu+m}} - 1 \right\} \tag{41a}$$

for constant stress boundary conditions, and

$$\gamma_A^c = \frac{\rho c \theta_A^o}{(1-\nu)\beta \tau_A^o} \left\{ \left[1 - \left(\frac{\ell_B}{\ell_A} \right)^{\frac{1}{m}} \left(\frac{\theta_B^o}{\theta_A^o} \right)^{\frac{\nu+m(1-\nu)}{m}} \right]^{\frac{(1-\nu)m}{\nu+m(1-\nu)}} - 1 \right\} \tag{41b}$$

for constant velocity boundary conditions; Eqn. (41b) holds only for $m \ll 1$.

3.2 Materials with Strain Hardening

Strain hardening cannot be ignored for most materials. In this section we derive analytical localization criteria for constitutive laws of the form

$$\tau = \mu(\theta)\ (\gamma + \gamma^o)^n\ \dot{\gamma}^m \tag{42}$$

where γ^o is the initial strain. The approach is similar to that used in Section 3.1.

3.2.1 Stress boundary condition

Elimination of γ between the constitutive law (42) and the energy equation (16) leads to

$$\frac{d\theta}{dt} = \frac{\beta\tau}{\rho C}^{\frac{m+1}{m}} \mu(\theta)^{-1/m} (\gamma + \gamma^{o})^{-n/m} . \tag{43}$$

We write (43) at two different points A and B, take account of the equilibrium equation (14), use (28) to eliminate γ, and integrate to obtain

$$\ell_A^{\frac{m+1}{m}} \int_{\theta_A^o}^{\theta_A} \mu(\zeta)^{1/m} \left[(\zeta - \theta_A^o) \frac{\rho C}{\beta\tau_A} + \gamma_A^o\right]^{n/m} d\zeta \tag{44}$$

$$= \ell_B^{\frac{m+1}{m}} \int_{\theta_B^o}^{\theta_B} \mu(\zeta)^{1/m} \left[(\zeta - \theta_B^o) \frac{\rho C}{\beta\tau_B} + \gamma_B^o\right]^{n/m} d\zeta .$$

Equation (44) is a generalization of (21) to strain hardening materials. Analysis of (44) analogous to that of (21), shows that L_∞ localization occurs if and only if the function

$$\theta \to \mu(\theta)^{1/m} \left[(\theta - \theta^o) \frac{\rho C}{\beta\tau} + \gamma^o\right]^{n/m}$$

is integrable at infinity. For the constitutive law

$$\tau = \mu_1 \theta^{\nu} (\gamma + \gamma^o)^n \dot{\gamma}^m \tag{45}$$

L_∞ localization occurs if and only if

$$\nu + n + m < 0 \ . \tag{46}$$

The inequality (46) provides a good illustration of the competition between the stabilizing effects of strain hardening ($n > 0$) and positive strain rate sensitivity ($m > 0$), and the destabilizing effects of thermal softening ($\nu < 0$). The localization criteria (46), obtained by MOLINARI and CLIFTON (1983), has also been obtained by FRESSENGEAS and MOLINARI (1986) as the criterion for the initial growth of a fluctuation based on a linear relative perturbation analysis. The inequality (46) differs from the condition

$$\nu + n < 0 \tag{47}$$

that must be satisfied for the initial growth of a fluctuation according to absolute linear perturbation analysis. The difference between the conditions (46) and (47) illustrates the tendency for absolute linear perturbation analysis to predict growth of fluctuations under some conditions for which the full nonlinear analysis predicts that localization will not occur.

For the constitutive law

$$\tau = \mu_o \, e^{-\alpha\theta} (\gamma + \gamma^o)^n \, \dot{\gamma}^m \tag{48}$$

a similar analysis shows that L_∞ localization occurs if and only if $\alpha > 0$ (thermal softening). For this constitutive equation the critical strain γ_A^C at A when the strain at point B becomes infinite is obtained by the substitution of (48) into (44) to obtain, after a change of variable,

$$K_A \int_{\gamma_A^o}^{\gamma_A^c} e^{-\nu} \nu^{n/m} d\nu = K_B \int_{\gamma_B^o}^{\infty} e^{-\nu} \nu^{n/m} d\nu \qquad (49)$$

where

$$\gamma_A^c = \frac{\alpha\beta\tau_A}{m\rho C}(\gamma_A^c + \gamma_A^o) \ , \ \gamma_A^o = \frac{\alpha\beta\tau}{m\rho C}\gamma_A^o$$

$$K_A = \ell_A^{\frac{1+n+m}{m}}$$

with A replaced by B for γ_B and K_B. Localization will occur at the point B, where the quantity on the right side of (49) is a minimum.

Equation (49) has the same form as that obtained by HUTCHINSON and NEALE (1977) in the study of the rupture of a viscoplastic bar in tension although the physical effects being modeled are different - their analysis included necking, but did not include the thermal softening which is included here.

3.2.2 Velocity boundary conditions

As in section 3.1 we consider constant velocity boundary conditions and assume that the strain rate sensitivity of the material is small (i.e. $m << 1$). To calculate the temperature from the energy equation (16) we replace $\dot{\gamma}$ by $\dot{\gamma}_o = V/h$ in the constitutive equation (42) and integrate to obtain

$$\int_{\theta^o}^{\theta} \frac{1}{\mu(\xi)} d\xi = \frac{\beta \dot{\gamma}_o^m}{\rho C (n+1)} \left[(\gamma + \gamma^o)^{n+1} - (\gamma^o)^{n+1} \right] . \qquad (50)$$

Substitution into (50) of functions $\mu(\theta)$ that model the temperature dependence of the flow stress gives the required relationship between the temperature θ and the shear strain γ. For $\mu(\theta) = \mu_1\theta^{\nu}$ we obtain

$$\theta(\gamma) = \theta^{o}\left[1 + (1-\nu)\frac{\beta\,\mu_1\,\dot{\gamma}_o^{m}}{\rho C\,(n+1)(\theta^{o})^{1-\nu}}\left[(\gamma+\gamma^{o})^{n+1} - (\gamma^{o})^{n+1}\right]\right]^{\frac{1}{1-\nu}} \qquad (51)$$

and for $\mu(\theta) = \mu_o\,e^{-\alpha\theta}$ we obtain

$$\theta(\gamma) = \theta^{o} + \frac{1}{\alpha}\log\left[1 + \frac{\alpha\,\beta\,\mu_o\,\dot{\gamma}_o^{m}\,e^{-\alpha\,\theta^{o}}}{\rho C\,(n+1)}\left[(\gamma+\gamma^{o})^{n+1} - (\gamma^{o})^{n+1}\right]\right]. \qquad (52)$$

These equations provide an approximate relationship between the temperature and the strain at each position as long as the exponent m is sufficiently small for the dependence of the shear stress on strain rate to be represented by $\dot{\gamma}_o^{m}$, where $\dot{\gamma}_o$ is the nominal strain rate, instead of by $\dot{\gamma}^{m}$, where $\dot{\gamma}$ is the local strain rate.

In order to investigate the critical conditions for localization, we substitute the functions $\theta(\gamma)$ obtained from (51) or (52) into the equation

$$\ell_A^{1/m}\int_0^{\gamma_A}\mu(\theta_A(\xi))^{1/m}\,(\xi+\gamma_A^{o})^{n/m}\,d\xi$$

$$= \ell_B^{1/m}\int_0^{\gamma_B}\mu(\theta_B(\xi))^{1/m}\,(\xi+\gamma_B^{o})^{n/m}\,d\xi\,. \qquad (53)$$

As before, L_∞ localization occurs if, and only if, the integral on the right side of (53) remains bounded as $\gamma_B \to \infty$. For $\theta(\gamma)$ given by Eqn. (51), the condition for L_∞ localization becomes

$$\nu + n + m\,(1-\nu) < 0 \tag{54}$$

for $0 < m \ll 1$, and $\nu < 1$. This condition is slightly more restrictive than the condition (46) obtained for stress boundary conditions. That is, the tendency for localization is slightly stronger for stress boundary conditions than for velocity boundary conditions in that the localization condition (46) is satisfied by all ν, m, and n which satisfy (54); however, for $m \ll 1$, the terms involving m in both (46) and (54) are often so small that, effectively, the localization conditions (46) and (54) are the same. For $\theta(\gamma)$ given by Eqn. (52) the condition for L_∞ localization is satisfied for all $\alpha > 0$ provided that m, n satisfy $m > 0$, $n > -1$.

4. Numerical Example

Dynamic torsion experiments for investigating shear localization have been performed by HARTLEY et al. (1986) on two different types of steel: CRS 1018 and HRS 1020. At the strain rates ($10^3 s^{-1}$) and temperatures ($\theta^0 = 300°K$) of these experiments the behavior of these materials can be represented reasonably well by a constitutive equation of the form (45). Numerical values of the various parameters in the model are given in Table 3 (SHAWKI (1986)). The strain γ^0 is taken to have the value 0.01 for both steels. More detailed fitting of the plastic response of these steels has been presented by KLEPACZKO (1986).

Table 3 Thermomechanical Properties of CRS 1018 and HRS 1020 Steels

Steel / Parameter	CRS 1018	HRS 1020
ν	- 0.38	- 0.51
n	0.015	0.12
m	0.019	0.0133
ρ	7800 kg/m^3	7800 kg/m^3
c	500 J/kg° K	500 J/kg° K
μ_1	3579 x 10^6 S.I.	7587 x 10^6 S.I.

Variations $\ell(y)$ in the wall thickness of the specimens were not reported by HARTLEY (1986). Subsequently, DUFFY (1986) has sectioned specimens used in such experiments to determine the variation in wall thickness, both along the length of the specimen and around its circumference. For CRS 1018 the wall

thickness is relatively uniform around the circumference, but strong variation - up to 10% - occur along the length of the specimen. For the purpose of this numerical example we take the geometrical factor (ℓ_B/ℓ_A) in the preceding analysis to be a parameter that varies from 0.9 to 0.999. In order to relate the critical strains γ_A^c to the nominal strain γ^c at localization (see Eqn. (32)), the variation in wall thickness $\ell_A = \ell(y_A)$ must be prescribed over the entire length of the specimen. Based on the general appearance of the sectioned specimens we take this variation to have the form

$$\frac{\ell(y)}{\ell_A} = 1 + \frac{\epsilon}{2}\left(\cos\frac{2\pi y}{h} - 1\right) \tag{55}$$

where ϵ is a geometrical parameter that is taken to vary from 10^{-1} to 10^{-6} to give the range of values of 0.9 to 0.999999 for ℓ_B/ℓ_A.

Boundary conditions for the dynamic torsion ("torsional Kolsky bar") experiment are effectively those of imposed constant velocity at the ends of the specimen. Hence, we use the solution for velocity boundary conditions given by Eqns. (51) and (53). The restriction to $m \ll 1$ that is required in obtaining (51) is well satisfied by the values $m = 0.019$ for the CRS and $m = 0.0133$ for the HRS. Evaluation of $\gamma^c(y_A)$ from (53) and integration over the length of the specimen, according to (32) gives the dependence of the critical strain γ^c on the geometrical imperfection parameter ϵ that is shown in Figs. 4 and 5. For small ϵ the nominal critical strain varies approximately as $\log \epsilon$, as predicted for the local critical strain by Eqn. (40b). In each figure the insert provides an expanded scale of the region of primary interest in the interpretation of torsional Kolsky bar experiments. For one value of ϵ ($\epsilon = 0.02$), the strain distribution at localization for CRS 1018 is shown in Fig. 6. The width of the band of intense shear (say, the

region for which $\gamma(y) > 3\gamma(0)$) is approximately 20% of the length of the specimen. Such relatively wide bands are observed in CRS 1018 (HARTLEY et al. (1985)). The predicted nominal stress-strain curves for the two steels, with the geometrical imperfection parameter ϵ equal to 0.02, are shown in Figs. 7 and 8; a corresponding curve for $\epsilon = 0.04$ is included in Fig. 8. The general features of the curves include a slowly rising segment during which the shearing is quite uniform, a slowly falling segment during which a broad band of enhanced shearing develops, and a sharply falling segment during which the shearing becomes intensely localized in a band. These general features are characteristic of the experimental records obtained in such experiments (HARTLEY, et al. (1986)). Numerical values for the strain at the peak of the stress-strain curve and the strain at the beginning of the sharp decline in stress are comparable to values obtained in experiments. However, the predicted rate of sharp decline is greater than normally measured. This rate of decline is affected by the detailed geometry of the initial imperfection which probably was not modeled adequately by the generic form (55) used to model the imperfection. Other difficulties with comparisons between theory and experiment in the steeply falling part of the curve include: (i) the inadequacy of the assumption that the stress obtained using the nominal strain rate can be used in calculating the local rate of energy dissipation, (ii) the likelihood that the final localization varies so strongly around the circumference of the specimen that a one-dimensional analysis is inappropriate; and (iii) the lack of constant velocity boundary conditions when the stress decreases strongly in torsional Kolsky bar experiments.

5. Conclusion

By assuming the deformation to be adiabatic and quasi-static, and by neglecting elasticity effects, we have characterized, analytically, the critical conditions for shear strain localization in simple shear. The assumed conditions are good approximations for the specimen sizes and strain rates that are commonly used in torsional Kolsky bar experiments on shear band formation in steels.

We assume the existence of initial inhomogeneities which are either geometrical defects or non-uniform fields of initial temperature or strain. The localization strain is obtained as a function of these defects, the material parameters and the boundary conditions. Two types of boundary conditions have been considered:

- constant applied stress
- constant applied velocity;

in the latter case, the analytical results are restricted to materials with weak strain rate sensitivity.

The results are particularly simple for materials without strain hardening. In this case, explicit expressions are obtained for the dependence of the critical strain on a defect parameter that characterizes the geometrical defect and the nonuniformity of the initial temperature. For materials with weak strain rate sensitivity the critical strain depends weakly (essentially logarithmically) on the amplitude of the imperfection for small imperfections.

Comparison of predictions of the theory with experimental results for a cold-rolled steel shows good agreement in the qualitative features of the response. Quantitative comparisons require detailed descriptions of the geometrical defects of the specimens used in the experiments. Preliminary

comparisons based on approximate representations of the geometrical imperfections of the specimens suggest that good quantitative agreement may be obtained once the defects are modeled accurately.

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Appendix A

Consider two piece-wise continuous functions g and h. The function g(x) is positive and nonintegrable at infinity, i.e. $\int_0^{+\infty} g(\zeta)\, d\zeta = +\infty$. If the function $h(\zeta)$ is equivalent to $g(\zeta)$ at infinity (i.e. $\lim_{\zeta\to\infty} h(\zeta)/g(\zeta) = 1$), then (DIEUDONNE (1968))

$$\int_a^x h(\zeta)\, d\zeta \sim \int_a^x g(\zeta)\, d\zeta \text{ , as } x \to \infty . \tag{A1}$$

With the choices $h(\zeta) = f(\zeta)^{1/m}$ and $g(\zeta) = a^{1/m}\, \zeta^{-p/m}$ we have, from Eqn. (4), $h \sim g$. Then Eqns. (3) and (A1) imply that for $m - p > 0$ we have

$$\ell_A^{1/m}\, a^{1/m} \frac{m}{m-p} \gamma_A{}^{(m-p)/m} \sim \ell_B^{1/m}\, a^{1/m} \frac{m}{m-p} \gamma_B{}^{(m-p)/m} \tag{A2}$$

since, when γ_A tends to infinity, γ_B must also tend to infinity. From (A2) it follows that

$$\lim_{\gamma_A\to\infty} (\gamma_B/\gamma_A) = (\ell_A/\ell_B)^{1/(m-p)} . \tag{A3}$$

Appendix B

During the localization process the local strain rate $\dot{\gamma}$ will differ from the nominal strain rate $\dot{\gamma}_o = V/h$. As a measurement of this difference we define the ratio

$$\lambda = \dot{\gamma}/\dot{\gamma}_o \; . \tag{B1}$$

We want to evaluate the acceptability of the approximation of replacing the stress

$$\tau = \mu \, e^{-\alpha \, \theta} \, \dot{\gamma}^m \tag{B2}$$

by the quantity

$$\tau_o = \mu \, e^{-\alpha \, \theta} \, \dot{\gamma}_o^m \tag{B3}$$

in calculating the heat generated by plastic working. For m = 0.01, the ratio $\tau/\tau_o = \lambda^m$ is bounded by $0.955 \leq \tau/\tau_o \leq 1.071$ for λ in the interval $[10^{-3}, 10^3]$. Thus, using the approximate stress τ_o leads to a maximum error of 7% for a variation in strain rate of six orders or magnitude.

List of Figures

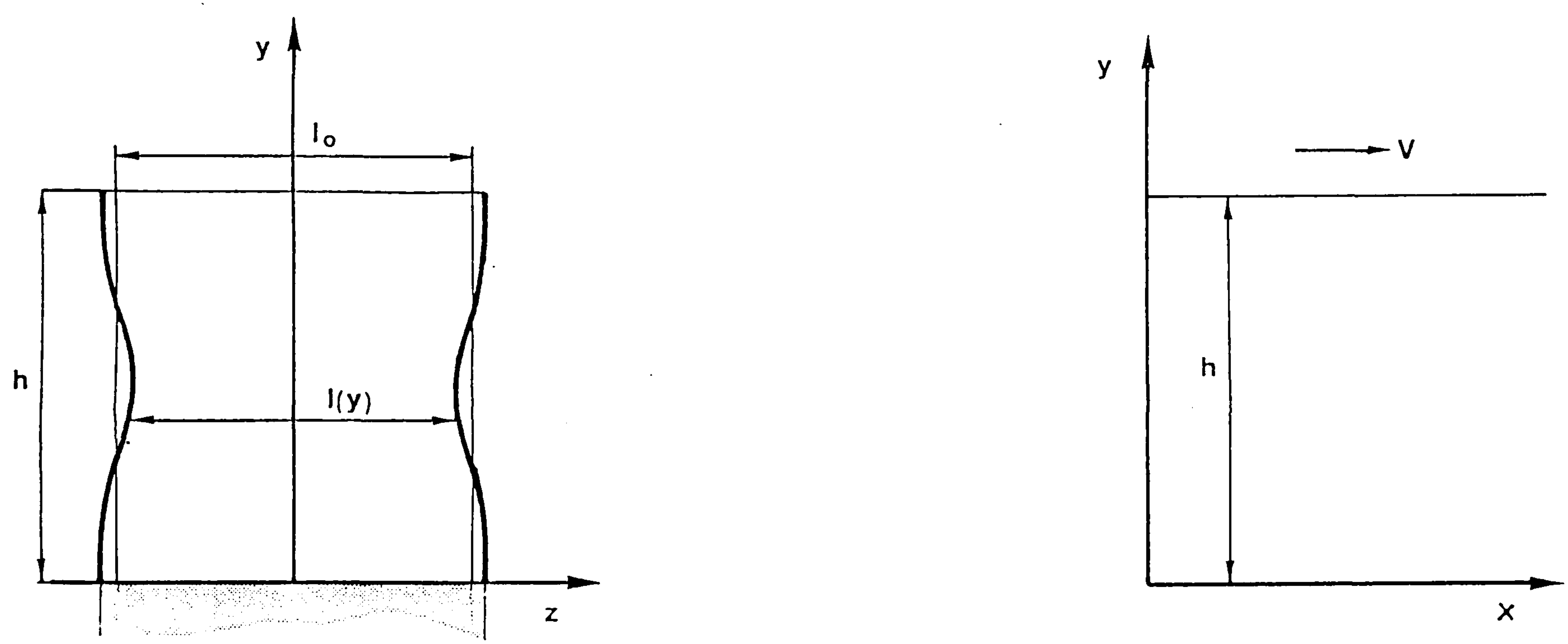


Figure 1

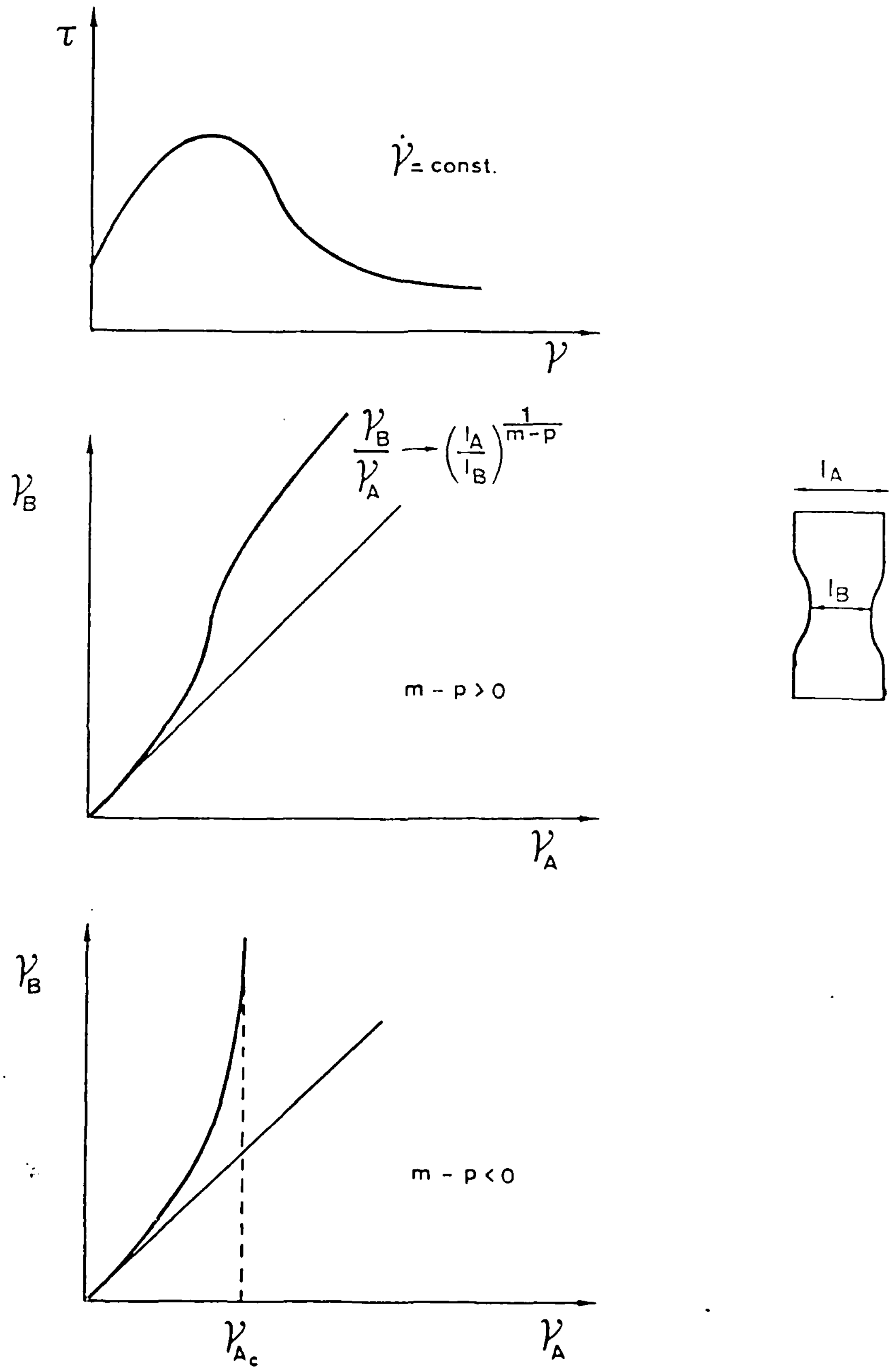
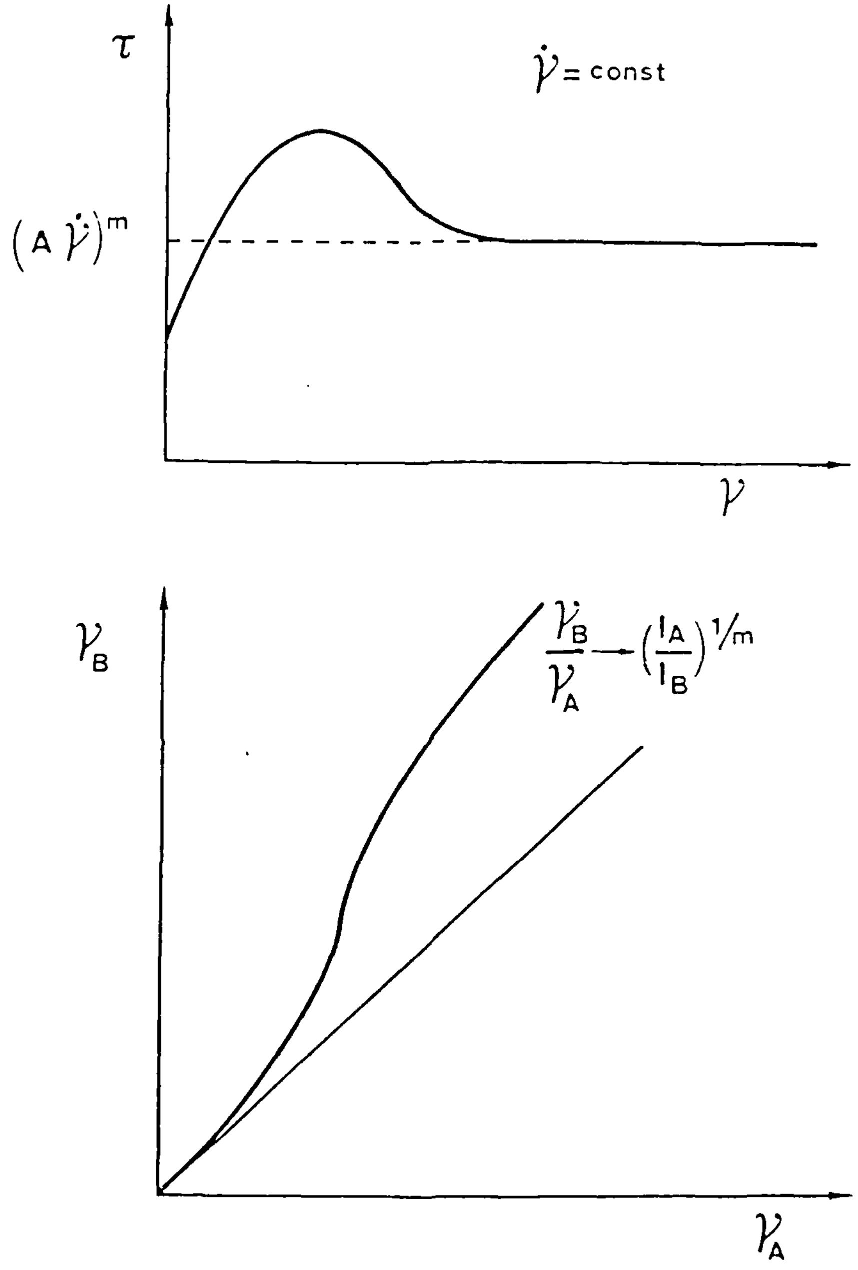
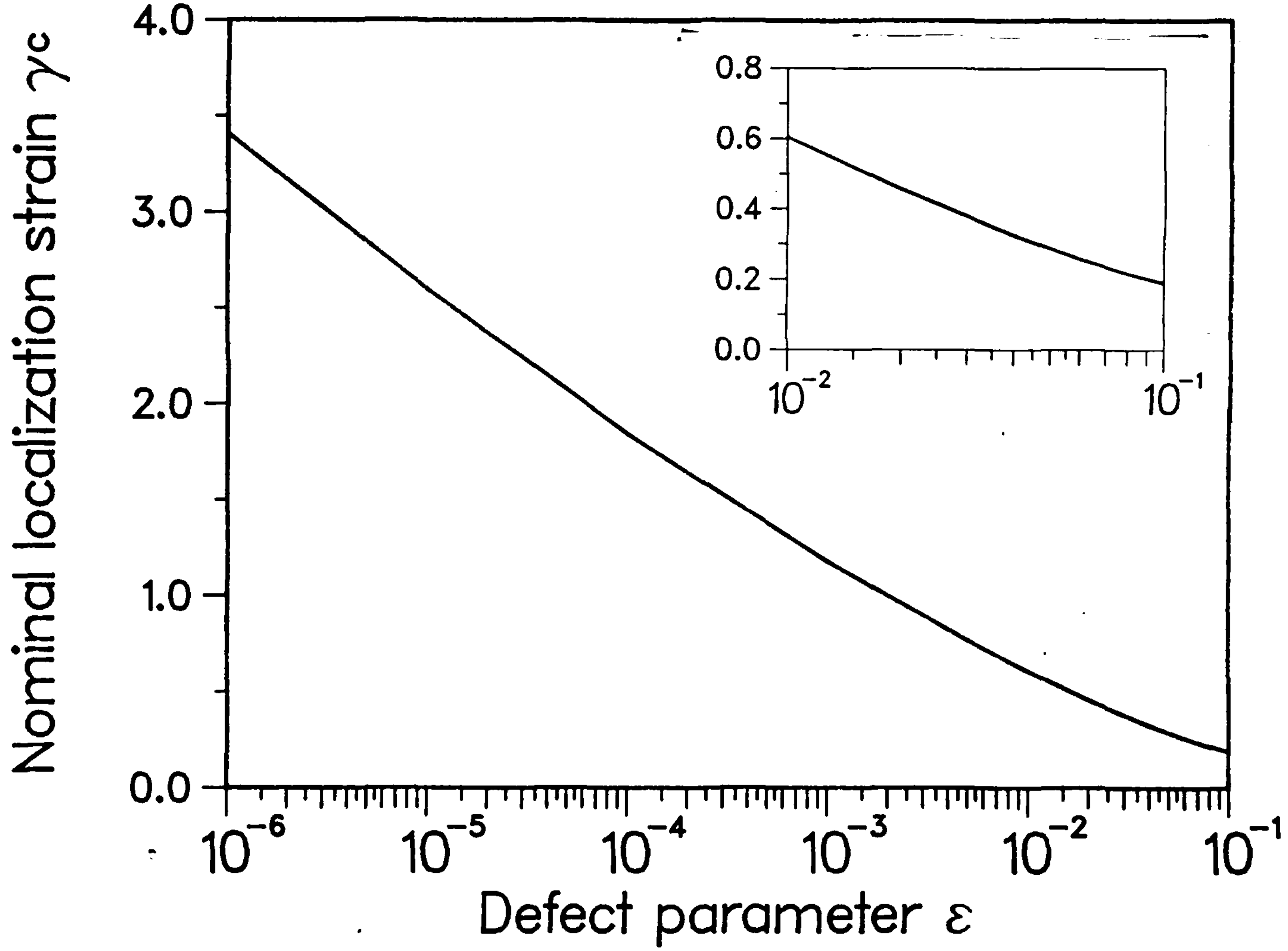


Figure 2



τ
$\dot{\gamma}$ = const
$(A\dot{\gamma})^m$
γ
γ_B
$\frac{\dot{\gamma}_B}{\gamma_A} \rightarrow \left(\frac{l_A}{l_B}\right)^{1/m}$
γ_A

Figure 3



Nominal localization strain γ^c
4.0
3.0
2.0
1.0
0.0
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Defect parameter ε
0.8
0.6
0.4
0.2
0.0
10^{-2}
10^{-1}

Figure 4

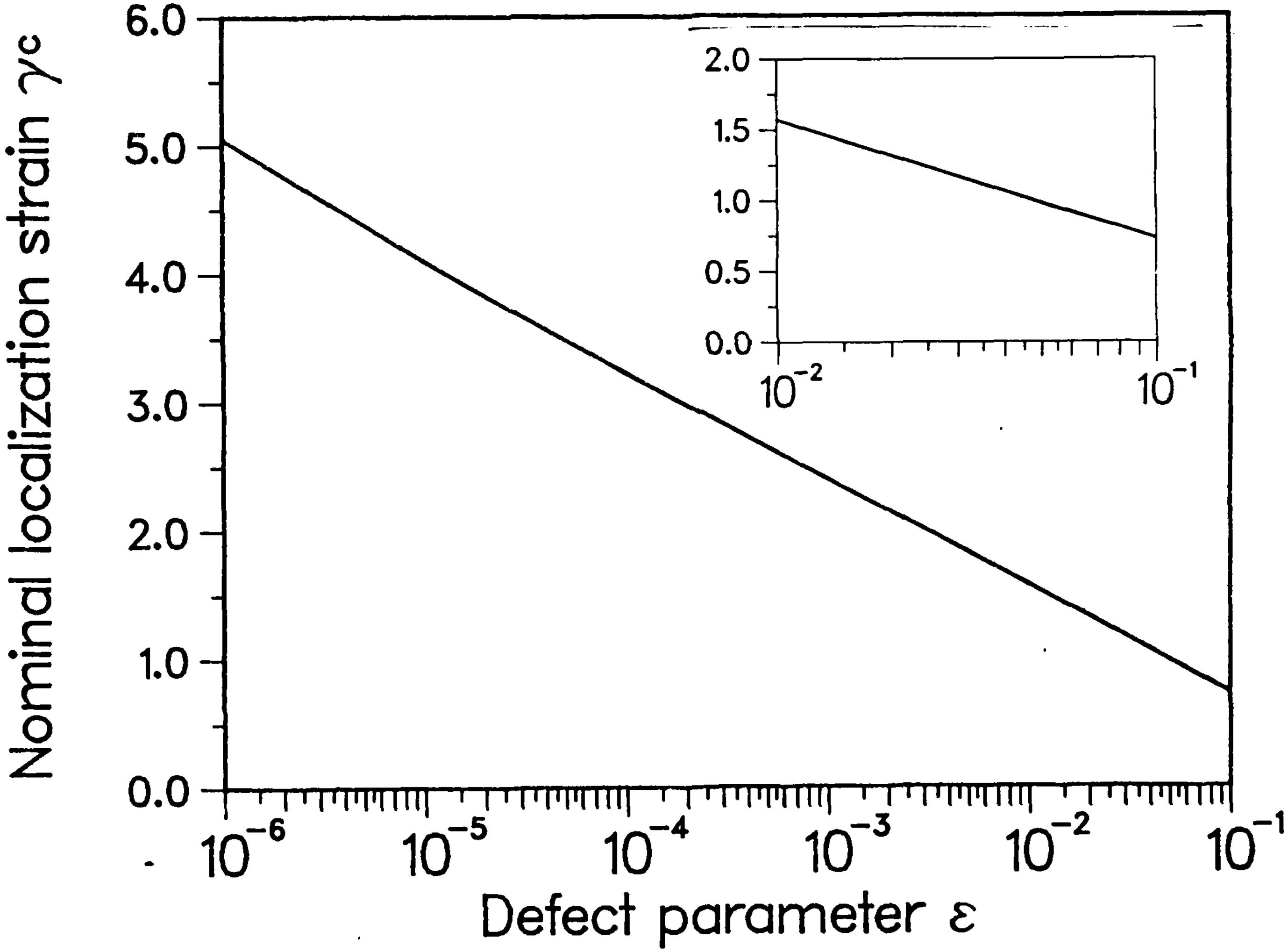

Nominal localization strain γ^c
6.0
5.0
4.0
3.0
2.0
1.0
0.0
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Defect parameter ε
2.0
1.5
1.0
0.5
0.0
10^{-2}
10^{-1}

Figure 5

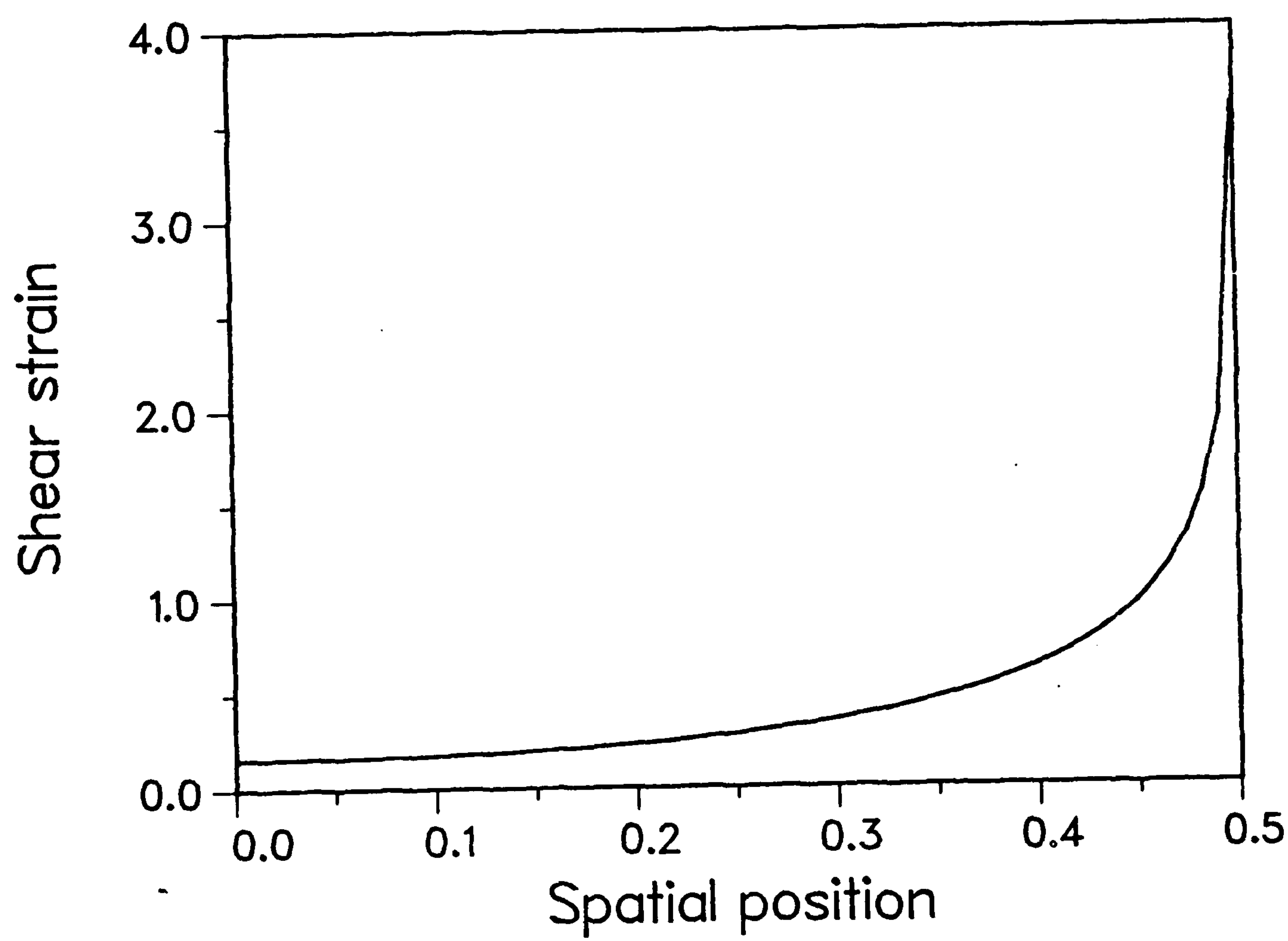


Figure 6

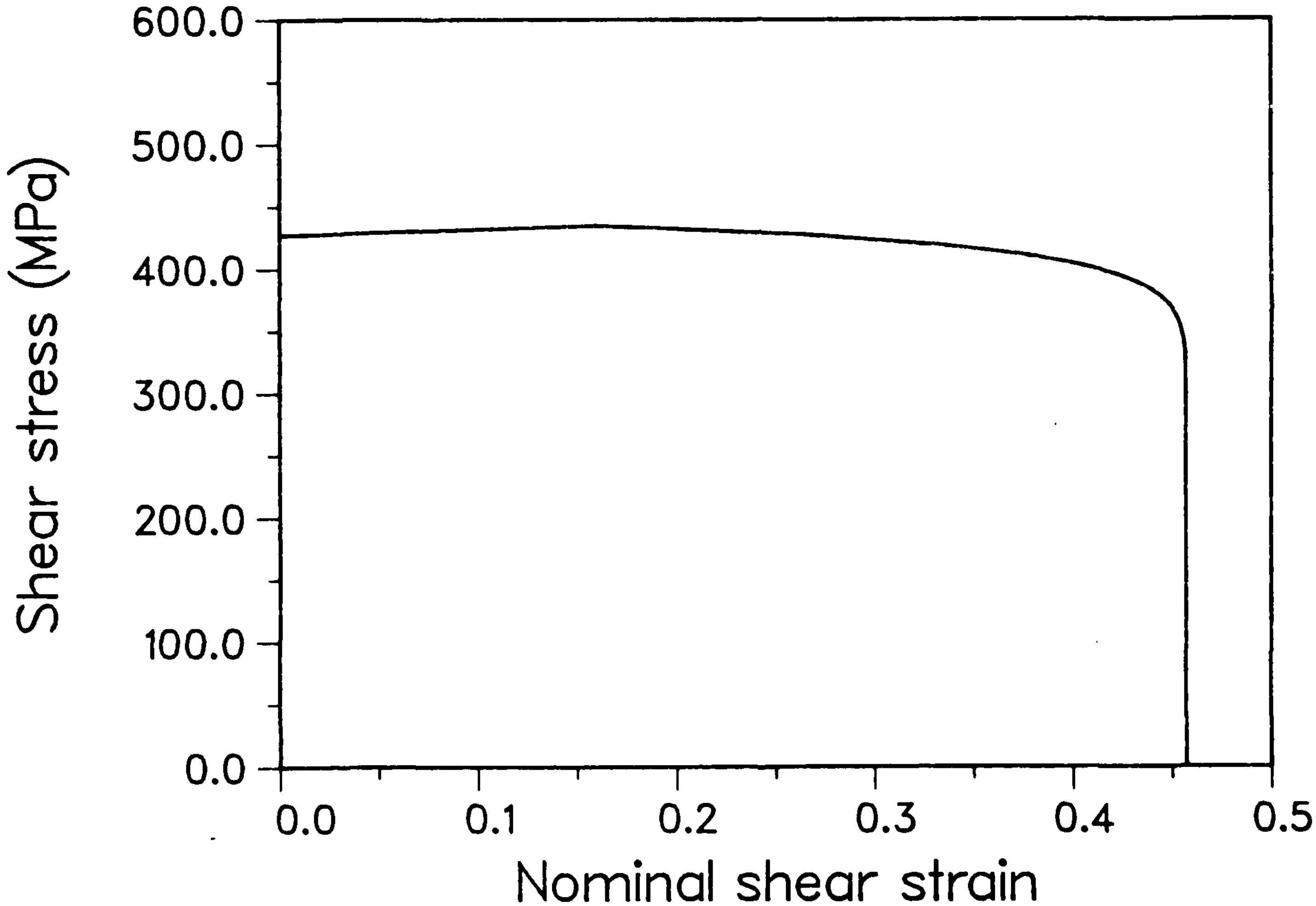
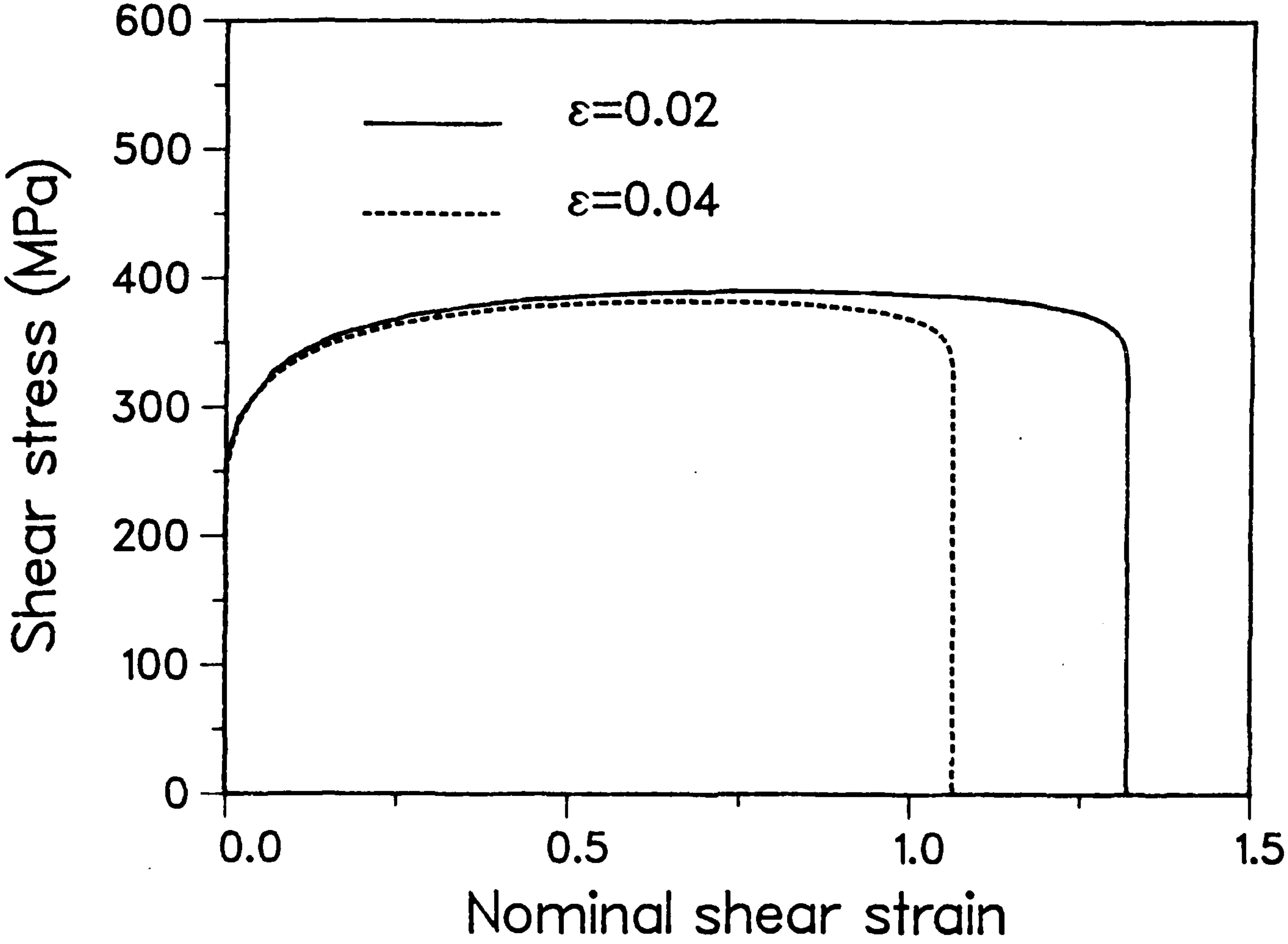

600.0
500.0
400.0
300.0
200.0
100.0
0.0
Shear stress (MPa)
0.0
0.1
0.2
0.3
0.4
0.5
Nominal shear strain

Figure 7



Shear stress (MPa)
600
500
400
300
200
100
0
ε=0.02
ε=0.04
0.0
0.5
1.0
1.5
Nominal shear strain

Figure 8